Drought Analysis Based on a Marked Cluster Poisson Model

ANA C. CEBRIÁN AND JESÚS ABAURREA

Departamento de Métodos Estadísticos, Universidad de Zaragoza, Zaragoza, Spain

(Manuscript received 9 March 2005, in final form 26 September 2005)

ABSTRACT

This paper presents an operational definition of drought events based on an "excess over threshold" approach applied on rainfall series and develops a stochastic model for describing droughts defined in that way. The model consists of a Poisson cluster process to represent drought occurrence and a marked process composed of three series of random variables (duration, deficit, and maximum intensity) to describe drought severity; it is theoretically justified, and adequate procedures to check its validity are suggested and applied on five Spanish rainfall series. Useful parameters for the design and planning of water resource systems, together with their confidence intervals, are estimated from the model.

1. Introduction

In a general way, a meteorological drought can be defined as a deficiency of precipitation resulting in a water shortage for some activities, over an area and during an extended period of time. Drought is a phenomenon that recurrently affects many regions in the world, such as the most southern and eastern areas of Spain, and prolonged droughts, although causing little structural damage, are critical for industrial, agricultural, and domestic water resources. In spite of the consequences, little drought planning is usually done, and water systems are often designed assuming an average year supply, which may lead to water shortage problems.

However, coping with droughts is possible through proper forecasting and planning. To reduce the impact of droughts, it is necessary to have a better knowledge of their characteristics to give answers to questions such as, How long we can expect a drought will last? How severe will it be? or How often will it recur? To answer these questions, the estimation of drought characteristics over a region is necessary and statistics of extremes can be helpful to obtain such estimations. That information would help planners to better estimate what is needed to survive a drought and to calculate, for example, the appropriate reservoir storage. Briefly, the description of drought characteristics by probability distributions and statistical models can improve the rigor of hydrological and climate applications and water resources management.

Thus, this paper has two main objectives: first, to establish an operational definition of drought flexible enough to be adapted to different climate characteristics and activities and, second, to develop a stochastic model that allows us to analyze the occurrence and severity of the droughts defined in that way; this work provides the necessary statistical methodology not only to estimate and make inferences on the proposed model but also to check its validity. This drought model will be also useful in other applications such as the study of trends due to a possible climate change (Abaurrea and Cebrián 2001) or the prediction of characteristics of the most severe drought to be observed in a given period of time (Abaurrea and Cebrián 2002). Also for a regional drought approach, consistent methods for at-site drought analysis are required (Hisdal and Tallaksen 2003).

In section 2 an operational definition of drought is established; in section 3, some extreme value theory results are summarized and the proposed drought model, a marked cluster Poisson process, is described; the description of the modeling process and the results of applying it on five Spanish rainfall series are presented in section 4 and final conclusions are given in section 5.

2. An operational definition of drought

Although the general concept of drought is clear, it is difficult to establish an operational definition able to reflect the differences caused by climate, regional char-

Corresponding author address: Ana C. Cebrián, Departamento de Métodos Estadísticos, Universidad de Zaragoza, Ed. Matemáticas, Pedro Cerbuna 12, Zaragoza 50009, Spain.
E-mail: acebrian@unizar.es

acteristics, and the activity involved. There are four main types of definitions for drought according to the aspects they consider: meteorological, hydrological, agricultural, and socioeconomic drought. Herein, we will focus on meteorological definition, but the suggested approach could be used for other definitions using adequate input information.

The meteorological drought definition is usually based on long-term precipitation departures from normal (approach followed in this study), though other climatic factors, such as high temperature, high wind, or low relative humidity can significantly aggravate the severity of the drought and are included in some definitions. A wide revision of drought definitions and indices classified according to the needed input information for their calculation can be found in Marachi (2000).

Another point that makes difficult to establish an operational drought definition is that, unlike other natural hazards, drought usually has a slow onset and develops over months or even years, and consequently, the starting and ending points are not clear. The analysis of droughts has been approached by different methodologies; for example, Sen (1976) and Moyé et al. (1988) use *run theory*, Kendall and Dracup (1992) use *renewal processes,* and Katz et al. (2002) provide a review on statistic methodology to analyze extremes in hydrological applications; in this study, we apply the *excess over threshold* (EOT) approach, also used by Zelenhasic and Salvai (1987), Madsen and Rosbjerg (1998), and Tallaksen et al. (1997).

The EOT approach leads to an operational definition that allows us to identify the beginning, the end, and the degree of severity of a drought. In this approach, a stochastic process related to precipitation, $s(t)$, is compared to a threshold, $U1(t)$, representing a critical level; a dry spell will occur when $s(t)$ is below $U1(t)$; see Fig. 1. However, this definition is not able to describe properly a drought event; in effect, drought is a phenomenon whose effects can persist for a long time, and during a lengthy dry period, the precipitation signal can slightly exceed the critical level for short periods of time without leading to the end of the drought. Such clustered dry spells must belong to the same drought as long as the inter–dry spell periods do not eliminate the effects of the water shortage. Thus, we define a *drought as a cluster of dependent dry spells*; the way of clustering the dry spells will be specified in section 4a.

To select an adequate signal $s(t)$, it was taken into account that drought is a phenomenon that requires a period of time to be established; thus, we use series where each observation, updated monthly, is the accumulated rainfall in the p previous months, where p will depend on the type of drought of interest. This signal is based on the same idea as the standard precipitation index (SPI), introduced by McKee et al. (1993), but is easier to calculate; the difference is that the SPI transforms the accumulated rainfall into a standard normal distribution scale, which is useful when the objective is the comparison of locations with different rainfall regimes. Herein, we will consider an accumulation period of one year (p = 12 months) that will reflect water deficiencies that affect processes depending on long-term precipitation, such as reservoir levels. Drought analysis for other fields, dry farming for example, should be based on series with a shorter accumulation period, but the suggested methodology would be still applicable, taking into account the seasonal character of those signals.

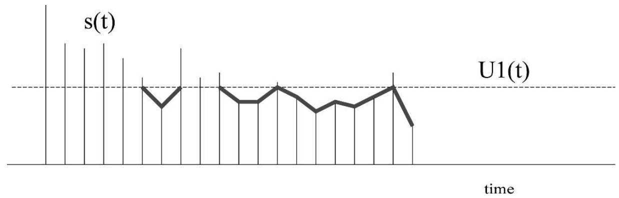

FIG. 1. Dry spell definition.

A *threshold* widely used for monitoring rainfall and preparing drought alerts is the tenth percentile of the rainfall series, $p10$ in short, proposed by Gibbs and Maher (1967); they demonstrated that the occurrence of the first decile in annual rainfall corresponded very well with droughts tabulated by Foley (1957), based on newspaper and other reports of effects on crops and livestock. The threshold used in SPI signals to detect severely dry periods is -1.5, which corresponds to the seventh percentile; in general, threshold values between the fifth and the tenth percentile can be considered adequate climate thresholds. Since our signal $s(t)$ has no seasonal component a constant threshold $U1$ can be used.

As for storms, floods, and other environmental disasters, more information about a drought is required than just its frequency; see Adamson et al. (1999) and Tallaksen et al. (1997). Consequently, we consider droughts as multivariate random events characterized by their duration or length, L; their peak or maximum intensity during the drought period, MI; and by the accumulated deficit to a normal rainfall value $U2$, D. In this analysis, we use as normal value the 30th percentile of the rainfall series. In our opinion, the deficit defined with respect to a normal rainfall value $U2$ instead of with respect to $U1$ (threshold defining the dry spells) represents the drought severity better than other definitions, such as the one by Zelenhasic and Salvai (1987), which does not take into account the exceedances of the inter-dry-spell periods in a cluster and is more realistic than

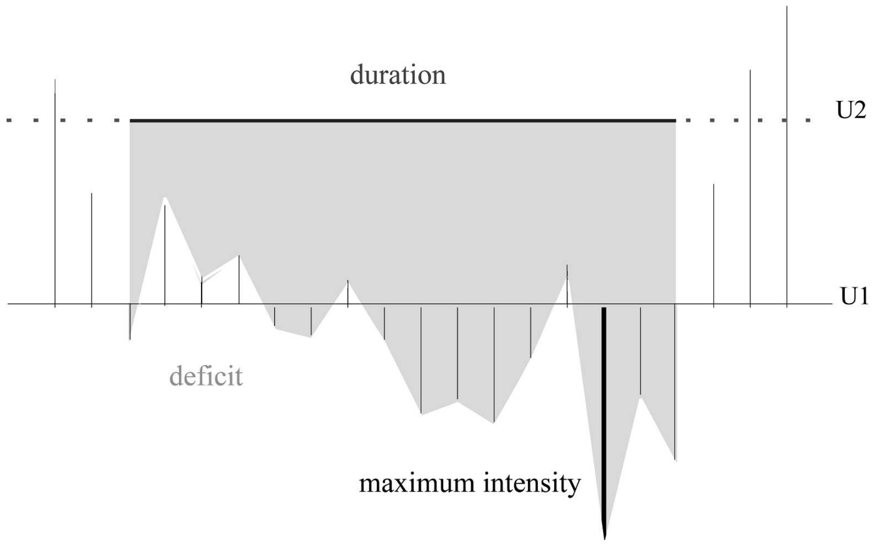

FIG. 2. Definition of the severity variables of a drought.

the one by Madsen and Rosbjerg (1998) that subtracts those exceedances. The three variables are illustrated in Fig. 2.

We remark that in drought analysis we deal with values under a threshold not over a threshold; that means that the observations of interest are $u - X|X \leq u$, not $X - u|X \geq u$, the most common in usual EOT approaches. This does not affect the process of the analysis once the series of occurrences and exceedances are formed.

3. Drought model: A marked Poisson cluster process

a. *Extreme value theory background*

The model suggested in this study is based on results from extreme value theory (EVT); see the monograph by Embrechts et al. (1997) and Davison and Smith (1990) for details on this topic. According to this theory, the occurrence process of the excesses of iid series converges, when extreme enough thresholds are used, to a homogeneous Poisson process and the distribution of the exceedances converges to a generalized Pareto (GP) distribution. Moreover, under certain conditions that are valid under the existence of short-range dependence but that require long-range independence of the data, it is proved that the excesses of stationary series have the same qualitative behavior. We recall that a homogeneous Poisson process (PP) on the interval $[0, \infty]$ is a stochastic point process; one characteristic property is that the recurrence times, the times between successive occurrence points, are independent exponential random variables. The GP distribution function is

$$G(x;\xi) = 1 - \left(1 + \xi\frac{x-\mu}{\sigma}\right)^{-1/\xi} \quad \begin{array}{ll} \text{for } (x-\mu)/\sigma \geq 0 & \text{if } \xi \geq 0 \\ \text{for } (x-\mu)/\sigma \in [0, -1/\xi] & \text{if } \xi < 0 \end{array},$$

where ξ is the so-called shape parameter and μ and $\sigma > 0$ are location and scale parameters. The shape parameter is dominant in determining its qualitative behavior: the distribution has an upper bound if $\xi < 0$, is unbounded if $\xi \geq 0$, and approaches an exponential distribution when $\xi \to 0$.

The previous theoretical results suggest that the occurrence of extreme dry spells can be modeled by a Poisson process. The dry spell process is not, strictly, a point process, since the events have a length; however, due to the fact that the recurrence times are large enough, the Poisson model can be a reasonable approximation by assigning an occurrence time to each spell. To check the robustness of results to the point selection, three possibilities were considered: the initial, the central, and the maximum intensity point of the event. Similar results were obtained for all of them, which confirms the validity of the approximation to a point process. Finally, we opted for using the maximum intensity point as occurrence time.

In section 2, a drought is defined as a cluster of dry spells; if, as EVT says, dry spells occur as PP, a reasonable model for representing the drought occurrence can be a Poisson cluster process. A Poisson cluster process (PCIP) is characterized by two properties:

- The occurrence process of clusters (droughts in this case) is a PP.
- The number of points (dry spells) in each cluster is an independent and identically distributed random variable that is also independent of the occurrence process of clusters.

The PCIP allows us to model the occurrence of drought events; to complete the model, random variables describing the characteristics of the drought severity (duration, deficit, and maximum intensity) are associated with each event. The union of both structures, a PCIP and iid random vectors associated with each occurrence, yields to the so-called marked Poisson cluster process (MPCIP).

b. *Justification of the Poisson character of the clustered events*

The EVT result previously mentioned justifies the Poisson character of the occurrence of the dry spells, but not the occurrence of the clusters since, in general, the arbitrary grouping of events in a PP does not preserve the Poisson character. However, in an EOT process, if the Poisson behavior of the occurrences is lost due to the clustering of the events, it is recovered by considering a more extreme threshold. More precisely, if a dry spell process is a PP, the occurrence process resulting from clustering its points will converge, when the threshold value becomes more extreme, to a PP. This is due to the fact that, with more extreme thresholds, the number of events per cluster converges to one or zero and the cluster process becomes a dry spell process; taking into account the Poisson character preservation under random thinning (see Cox and Isham 1980), the asymptotic Poisson character of the cluster process is obtained.

4. Modeling process

In this section, we describe the modeling process of rainfall series using the suggested approach and, to confirm its validity, we applied it on some Spanish series.

Five rainfall series are analyzed: Burgos, Daroca, Huesca, Madrid, and Murcia; they have been selected among the longest available series in Spain, in an attempt to get a representative network on the eastern part of the country. The series present a similar climate behavior with a bimodal pattern with two dry seasons, in summer and winter. All the series start in 1901 (except Daroca, which starts in 1910) and finish in 1994 (except Huesca and Daroca, which finish in 1998). Quality and homogeneity of the five series were studied by González-Rouco et al. (2001) and the inhomogeneities detected in the Burgos and Huesca series were corrected according to their conclusions.

a. *Cluster definition*

The first issue to deal with in the modeling process is how to define the clusters of dependent dry spells. Davison and Smith (1990) discuss and compare some methods for identifying clusters, including parametric ones (Neyman–Scott, Bartlett–Lewis, and Markov-chains models). They conclude that according to the results obtained by different authors, little seems to be gained by fitting parametric models instead of applying empirical rules.

Concerning empirical criteria, the most widely used is run declustering (Leadbetter et al. 1983); this approach assumes that two events belong to different clusters if they are separated for a long enough period where the signal is below the threshold level. The main problem with this criterion is the choice of the separating length, largely arbitrary, which can significantly influence later estimations.

Tallaksen et al. (1997) describe and compare three different pooling procedures for hydrological droughts. One of them, the inter-event time and volume criterion (IC), is based on the aforementioned run declustering but also includes information about the water volume generated during the inter–dry spell periods in a cluster; this kind of criterion is also used by Rasmussen et al. (1994). Ferro and Segers (2003) develop an automatic declustering scheme that relies on a prior estimation of the extremal index of the process; its main advantage is that cluster identification does not depend on any arbitrary choice.

After trying out several criteria we used an empirical rule based on the Ferro and Segers (2003) procedure but also including information about the intensity of the signal during the inter–dry spell periods: two dry events belong to the same cluster (drought) if

- the time between their occurrence is less or equal to $T_{(C)}$, the value obtained with the Ferro and Segers procedure;
- no intensity value during the time interval between them reaches a normal rainfall value $U2$ = 30th percentile. We consider this threshold value enough to

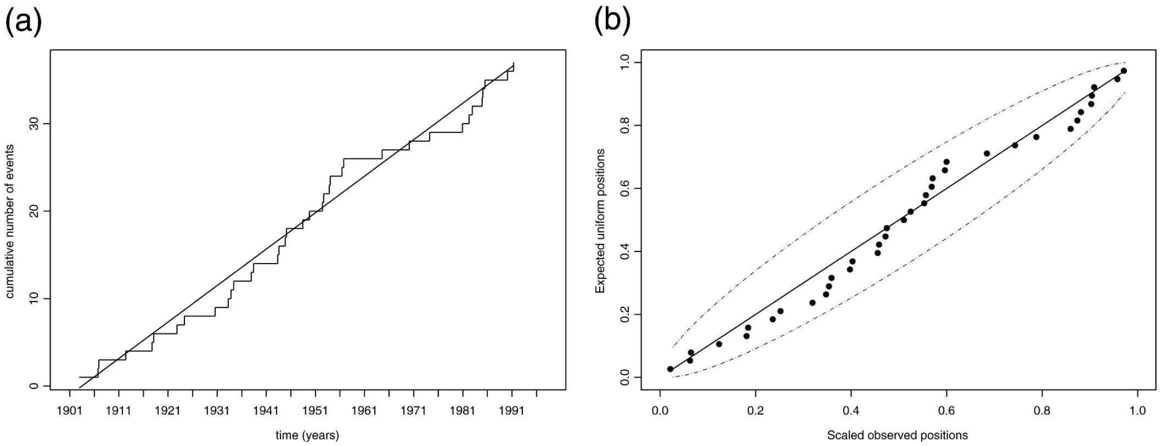

FIG. 3. Cumulative number of (a) *ds*10 and (b) uniform qqplot of the position times of the *ds*10 occurrences; Madrid, *ds*10 series.

assure that the system recovers from the previous drought effects, but it can be changed to more appropriate values for other applications.

An example of a cluster of dry spells defined in this way can be seen in Fig. 2. The values of $T_{(C)}$ for the different series range from 2 to 5 months.

b. Checking the hypothesis of the model

In this section, tools for checking the model are reviewed. We start by mentioning some plots designed to check the extreme character of a process in terms of the threshold value. Coles (2001) provides a review of tools for threshold selection such as the mean residual life or the stability plot, both based on the fit of a GPD to the exceedances of the process. The Gertensgarbe procedure (Gertensgarbe and Werner 1989) also aims to determine the starting threshold of the extreme value region of the exceedances.

Given a fixed threshold, a fast control of the homogeneous Poisson character of the occurrence process is provided by the plot by Castro and Pérez-Abreu (1994), based on the probability generating functional. But if a more exhaustive control is wanted, it must be verified that the series resulting from the considered threshold satisfy all the hypotheses required by the model: Poisson character of the occurrence, homogeneity, iid of the number of events per cluster and iid of the marked series. The tools for these analyses are detailed in appendix A.

RESULTS

(i) Dry spell process

We start by analyzing the occurrence series of the dry spells defined with the *p*10 threshold, denoted by *ds*10, since the justification of the Poisson character of the droughts (clusters of dry spells) is based on the Poisson behavior of the underlying dry spell process.

For all the *ds*10 series the Poisson control is satisfactory according to the graphical tools and the exponential Kolmogorov–Smirnov test; consequently, we can assure the Poisson behavior of the dry spells corresponding to *p*10 and, by the properties of a PP, to any more extreme threshold. Abaurrea and Cebrián (2001) found some fluctuations in the mean rainfall level for some Spanish series, but no indication of inhomogeneity is detected in the dry spell series according to the exhaustive analysis performed (see appendix A for details); as an example, we can see in Fig. 3 the linearity of the cumulative number of events and the uniform character of the observed positions of the dry spell occurrence in Madrid.

The need to use a clustered model is also confirmed since, as expected, the dry spells are found to be dependent: concerning the occurrence, independence can be assumed but a significant dependence (correlation and lack of randomness) appears in most of the deficit and maximum intensity series (see Table 1).

(ii) Drought process: Occurrence

Once the *ds*10 series are analyzed, we study the behavior of the series after the clustering process, that is, the drought series (*dr*10 in short). Using the same controls as before we conclude that Burgos, Huesca, Daroca, and Murcia *dr*10 series maintain the Poisson behavior after the clustering (see Table 2 for a summary of the results); only in Madrid it is lost according to the exponential qqplot (see Fig. 4) and the significant Kolmogorov–Smirnov *p* value, 0.04. But, as stated in section 3b, the Poisson behavior is recovered using a more extreme threshold, the sixth percentile, *p*6, in this

TABLE 1. Null autocorrelation and randomness tests of the severity variables corresponding to *ds*10.

	Burgos	Daroca	Huesca	Madrid	Murcia
L: *p*-value Kendall null autocorrelation	0.75	0.33	0.16	0.62	0.75
L: *p*-value run test	0.49	0.30	0.06	0.35	0.27
D: *p*-value Kendall null autocorrelation	0.12	0.18	0.06	0.16	0.93
D: *p*-value run test	0.85	0.00	0.04	0.09	0.69
MI: *p*-value Kendall null autocorrelation	0.18	0.10	0.03	0.06	0.78
MI: *p*-value run test	0.45	0.02	0.04	0.09	0.29

case; for *dr*6 the Kolmogorov–Smirnov *p* value becomes nonsignificant and the qq plot linear (see Fig. 4). The clustering process keeps the homogeneity of the process. Neither the occurrence nor the severity series show dependence, with all the null correlation and randomness tests being insignificant (see Table 2).

(iii) Drought process: Marks describing severity

Concerning the independence and identical distribution of *L*, *D*, and MI series, satisfactory results are obtained. Only some graphical indications of seasonal behavior are observed in Madrid duration and deficit series: according to the bubble plot in Fig. 5, where droughts are represented using bubbles of size proportional to their length (see appendix Ab for details), it seems that the longest droughts are those that start between January and April; however, statistical tests are not significant even for this location. The hypothesis of independence of the number of events per cluster series is also not rejected according to the null autocorrelation Kendall test.

To sum up, we can say that

- Burgos, Daroca, Huesca, and Murcia *dr*10 processes follow an MPCIP; and
- the Madrid *dr*6 process also fits an MPCIP.

c. Estimating the model

The estimation is performed separately for the two parts of the model. For the occurrence process, only the Poisson parameter λ has to be estimated. Concerning the marked process, for each one of the three severity variables, we have to choose an adequate distribution and estimate their parameters; according to EVT results from section 3a, it is expected that maximum intensity of droughts follows a GP distribution, but there is no theoretical result concerning the distribution of the deficit and the length. Thus, for each variable, we perform an empirical analysis to select the best probability distribution from a wide range of positive distributions. All the estimations are performed by maximum likelihood; see appendix B for more details.

Once the basic parameters of the model are estimated, other useful values in water resource planning such as expected values, medians, and high percentiles can be obtained from the model. In particular, we can calculate the expected value of the recurrence times as $E(R_t) = 1/\lambda$ since, in a PP, the time between two occurrences is exponential with mean equal to the inverse parameter of the intensity parameter λ of the process. The expected number of droughts in T yr ($= 12T$ months) can also be obtained given that, in a PP, the number of occurrences in a given period of time has a Poisson distribution of mean λ multiplied by the length of the period, $E(N_T) = 12T\lambda$.

Another important measure in hydrology is the *return value* of a magnitude in a period of length t, V_t, which represents the value of the magnitude that is expected to be exceeded on average once in that pe-

TABLE 2. Results of the check analysis of the Poisson model; all *dr*10 series and *dr*6 for Madrid.

	Burgos *dr*10	Daroca *dr*10	Huesca *dr*10	Madrid *dr*10	Madrid *dr*6	Murcia *dr*10
Threshold *U*1	4010	3060	4030	3080	2900	1630
No. of droughts	25	23	25	26	20	17
Poisson character *p*-value KS (exponential)	0.22	0.23	>0.38	0.04	>0.38	0.22
		Independence				
p-value Kendall null autocorrelation	0.16	0.81	0.63	0.88	0.82	0.84
p-value run test	0.41	0.65	0.68	0.54	0.81	0.62
		Homogeneity				
p-value seasonal behavior test	0.57	0.84	0.50	0.07	0.10	0.33
p-value Military Handbook test	0.93	0.38	0.52	0.69	0.44	0.71
p-value Laplace test	0.24	0.91	0.84	0.62	0.87	0.85

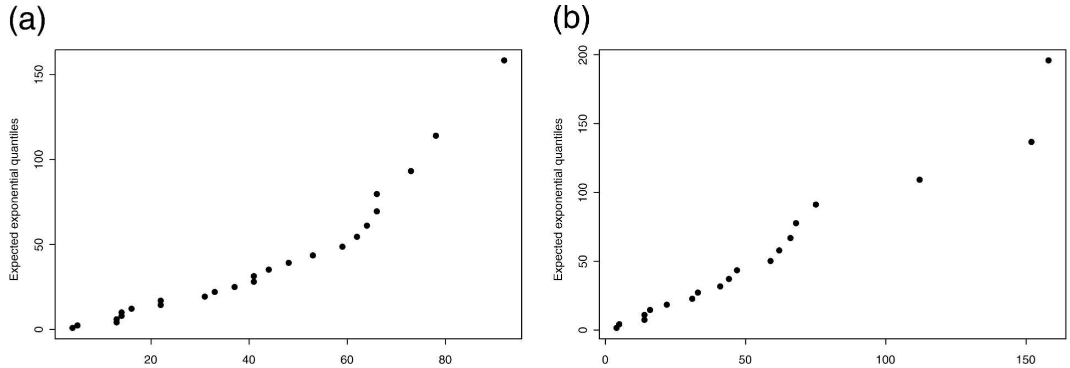

FIG. 4. Exponential qq plots of the drought recurrence times with threshold (a) $p10$ and (b) $p6$; Madrid.

riod; equivalently, it is the value that is exceeded in such a period with probability $1/t$. Given a PP of intensity λ whose occurrences have associated a variable X with distribution function F, to calculate the return value V_t of X we have to take into account that the expected number of exceedances in a period of length t is λt and the expected number of observations of X exceeding a value x will be $\lambda t[1 - F(x)]$. The value that makes this expression equal to 1 will be the return value; that is $1 - F(V_t) = 1/(\lambda t)$ or equivalently,

$$V_t = F^{-1}\left(1 - \frac{1}{\lambda t}\right).$$

Thus, the return value of any of the severity variables can be easily estimated from the Poisson parameter and the corresponding estimated distribution function F.

The variability of the predictions cannot be neglected and, in practice, variability measures and confidence intervals are as important as the point estimations. We calculate them using some asymptotic properties of the maximum likelihood estimators (MLE) and bootstrap methods; see appendix B for more details.

RESULTS

(i) Drought process: Occurrence

In Table 3 we show, together with their confidence intervals, the MLE of λ, the expected value of the recurrence times $E(R_t)$, and, as an example, the expected number of droughts in 100 yr, $E(N_{100})$. The expected recurrence times range from a bit more than 3 yr (38.5 months in Burgos) to more than 4 yr (almost 53 months in Madrid and Murcia). The results show that with this threshold definition, which depends on climate characteristics of the location, Murcia, the driest location, presents a low number of droughts and Burgos presents the highest drought frequency.

(ii) Drought process: Marks describing severity

In the five series analyzed, we find that duration fits a no memory distribution, a shifted exponential (exponential distribution has been shifted one unity since it takes values greater than 0 but duration is always greater than 1); the geometric distribution could be also used if a discrete representation is preferred. The deficit is properly fitted by a gamma distribution and, in

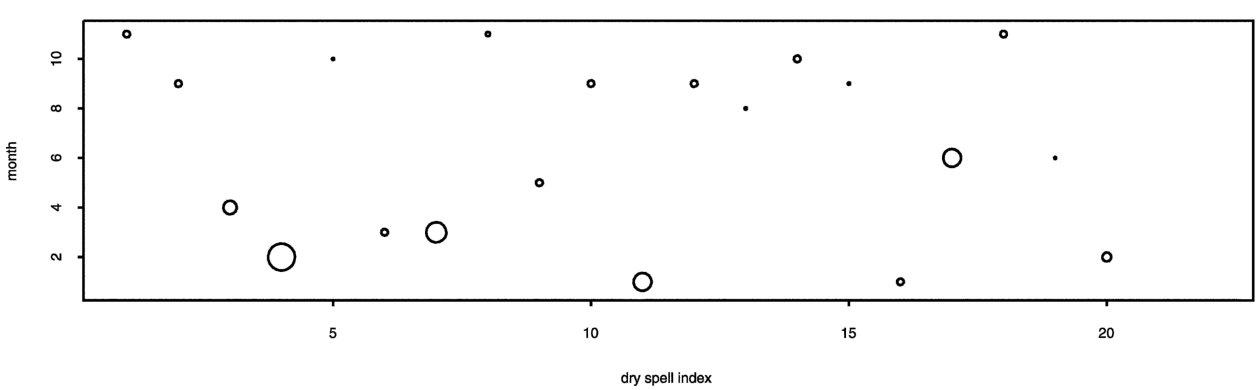

FIG. 5. Bubble plot of drought duration; Madrid.

TABLE 3. Estimated parameters of the drought occurrence process.

	Burgos *dr*10	Daroca *dr*10	Huesca *dr*10	Madrid *dr*6	Murcia *dr*10
$\hat{\lambda}$ CI (95%)	0.026 (0.017, 0.037)	0.023 (0.015, 0.033)	0.023 (0.015, 0.033)	0.019 (0.012, 0.028)	0.019 (0.011, 0.029)
$E(R_t)$ (months) CI (95%)	38.5 (27.1, 58.9)	43.5 (30.0, 68.6)	43.5 (30.4, 67.2)	52.6 (35.5, 86.2)	52.6 (34.4, 90.3)
$E(N_{100})$ CI (95%)	31.2 (20.4, 44.3)	27.6 (17.5, 40.0)	27.6 (17.9, 39.4)	22.8 (13.9, 33.8)	22.8 (13.3, 34.8)

some cases, lognormal or exponential distributions are also good alternatives. The best fit for maximum intensity, as expected from EVT, is provided by the GP distribution; *p* values of the goodness-of-fit tests described in appendix B are shown in Table 4.

Estimation of some useful values for the three variables, together with their confidence intervals, are summarized in Table 5. We can see that Murcia presents the longest and largest mean deficit droughts; the most intense ones appear in Daroca, which also presents high *p*99 values both in intensity and deficit. On the other hand, in Burgos and Madrid the droughts tend to be shorter. We observe that there is a high rank correlation between the *p*99 of the distribution and the return values in 100 yr (length of this period, 1200 months); these values are useful to design facilities to assure the water supply during the most severe drought periods in a given period of time. For example, it can be expected that in Madrid only 1% of the droughts will last more than 18 months and the return value of the deficit in 100 yr will be greater than 10.000 dl m^{-2}.

5. Conclusions

As pointed out throughout this work, there is a need to improve the study of the characteristics of meteorological and hydrological droughts, since a better knowledge of the status and dynamics of water resources will be helpful in making decisions concerning the management and design of water facilities. Thus, the objective of this work is, first, to establish an operational definition of drought and, then, to develop a stochastic model to represent the occurrence and severity of this phenomenon.

We define a drought event as a cluster of dependent dry spells, resulting from applying the "excess over threshold" approach on rainfall series using an adequate climate threshold; we describe drought severity by three variables: length, maximum intensity, and deficit to a normal rainfall value.

The use of a cluster Poisson process marked by the three variables describing severity is found to adequately represent droughts defined in that way, since climate thresholds (between the tenth and fifth percentiles) are extreme enough to get the Poisson behavior. The validity of this model is justified by asymptotic results of EVT and confirmed through a thorough check analysis of five Spanish rainfall series. Concerning the severity series, it is found that exponential, gamma, and GP distributions fit the duration, the deficit, and the maximum intensity, respectively. No significant time trend during the last century is detected either in the occurrence or the marked processes.

To sum up, the drought definition and model proposed in this work enable a deeper analysis of the occurrence and severity of droughts; not only expected values but also high percentiles of drought frequency and severity and the widely used return values, together with their confidence intervals, are reliably estimated.

Finally, we want to point out that this approach can be used to model different types of droughts or even other extreme climate phenomena, using adequate signals. Future work involves the fitting of nonhomogeneous Poisson processes; this generalization of the

TABLE 4. *p*-values of Kolmogorov–Smirnov goodness-of-fit tests for the severity variables: *L* (shifted exponential), *D* (gamma), and MI (GP).

	Burgos *dr*10	Huesca *dr*10	Daroca *dr*10	Madrid *dr*6	Murcia *dr*10
		Length			
p-value KS (uniform) for $\hat{F}$	0.21	0.32	0.51	0.27	0.29
p-value KS (exponential)	0.16	0.18	0.50	0.11	0.21
		Deficit			
p-value KS (uniform) for $\hat{F}$	0.32	0.41	0.40	0.94	0.59
p-value KS (gamma)	0.18	0.30	0.21	0.74	0.44
		Maximum intensity			
p-value KS (uniform) for $\hat{F}$	0.67	0.53	0.77	0.83	0.66

TABLE 5. Estimated parameters of the severity variables.

	Burgos *dr*10	Huesca *dr*10	Daroca *dr*10	Madrid *dr*6	Murcia *dr*10
			Length (months)		
$\hat{\lambda}_L$	0.23 (0.15, 0.32)	0.20 (0.13, 0.28)	0.18 (0.1, 0.3)	0.25 (0.2, 0.4)	0.14 (0.1, 0.2)
Mean	4.4 (3.1, 6.8)	5.1 (3.6, 7.8)	5.6 (3.8, 8.8)	4.0 (2.7, 6.5)	7.1 (4.6, 12.1)
*p*50	3.1 (2.2, 4.7)	3.5 (2.5, 5.4)	3.9 (2.7, 6.1)	2.8 (1.9, 4.5)	4.9 (3.2, 8.4)
*p*99	20.4 (14.3, 31.2)	23.4 (16.4, 36.1)	25.6 (17.7, 40.4)	18.4 (12.4, 30.2)	32.5 (21.3, 55.8)
V_{100}	15.2	16.8	18.5	12.5	22.1
			Deficit (dl m^{-2})		
$\hat{\alpha}$	1.2 (0.7, 1.7)	1.3 (0.7, 1.8)	1.3 (0.7, 1.8)	2.0 (0.9, 3.1)	1.7 (0.7, 2.7)
$\hat{\beta}$	4570 (2662, 6477)	4018 (2167, 5869)	4965 (2754, 7175)	2076 (785, 3367)	3629 (1380, 5878)
Mean	5387 (3854, 7107)	5108 (3663, 6766)	6405 (4722, 8349)	4178 (3142, 5326)	6167 (4477, 8044)
*p*50	3963 (2579, 5789)	3848 (2427, 5734)	4847 (3159, 6923)	3510 (2436, 4964)	5009 (3253, 7374)
*p*99	22862 (10 991, 27 026)	20896 (9780, 24 228)	26017 (12925, 30987)	13831 (7088, 16 127)	22 017 (10 257, 24 958)
V_{100}	17 352	15 449	19 265	10 220	15 964
			Maximum intensity (dl m^{-2})		
$\hat{\xi}$	−0.63 (−1, −0.26)	−0.25 (−0.61, 0.11)	−0.25 (−0.68, 0.18)	−0.49 (−1.15, 0.18)	−0.74 (−1.1, 0.53)
$\hat{\sigma}$	810 (424, 1197)	509 (237, 780)	724 (310, 1138)	629 (205, 1054)	763 (196, 1330)
Mean	497 (383, 607)	407 (295, 527)	578 (420, 739)	423 (318, 530)	440 (328, 544)
*p*50	455 (296, 636)	324 (197, 495)	460 (282, 694)	370 (237, 529)	414 (244, 595)
*p*99	1216 (942, 1234)	1391 (824, 1498)	1972 (1191, 2126)	1152 (779, 1175)	1002 (744, 1006)
V_{100}	1139	1147	1627	1008	933

model will allow us to analyze the possible influence of some covariables on the occurrence and severity of the droughts and the modeling of series affected by climate change.

Acknowledgments. The authors want to thank CICYT (Spain) for financial support (project REN2002-00009).

APPENDIX A

Tools for Checking the Model

a. Occurrence process

1) POISSON CHARACTER

The Poisson character of a process can be validated using any of the characterizations of a PP [see Cox and Isham (1980) for different characterizations], but we found that controlling the exponential character of the recurrence times provides the most reliable results, better than, for example, controlling the Poisson distribution of the number of events in a time interval. Thus, an exponential distribution is fitted to the recurrence times by maximum likelihood, and its goodness of fit is verified using an exponential qqplot and the Kolmogorov–Smirnov test (KS).

2) INDEPENDENCE

To check independence of the recurrence times, the correlogram is analyzed and null autocorrelation contrasted using tests based on Kendall and Spearman coefficients. Randomness is checked using the run test, the increasing and decreasing run test, and the sign test; details on these tests can be found in Gibbons and Chakraborti (1992).

3) HOMOGENEITY

Several aspects are controlled to check this property:

1) *Seasonal behavior.* To check a possible seasonal behavior, we divide the year into six 2-month periods (January–February, March–April, . . . , November–December) and a uniformity χ^2 test and a plot of the proportion of number of events by period are performed.
2) *Trends.* The existence of a monotone trend in the series of recurrence times is checked by applying a test to contrast the null value of the Kendall correlation coefficient between the recurrence times and their order number; we also use specific tests for PP, such as the Military Handbook test and the Laplace test, designed to contrast homogeneity (no trend) against potential and loglinear trends, respectively; see Meeker and Escobar (1998) for more details.

Given the importance of the hypothesis of homogeneity, we suggest some additional tools. In a homogeneous PP the expected number of events in an interval (0, t) must be a linear function of t, and consequently, the linearity of the accumulated number of events can be graphically analyzed. A homogeneous PP also verifies that if, in a given period of time, n events are observed, their occurrence times are distributed as independent and uniform variables in that period. This property can be controlled using the KS test and a uniform qqplot with a beta confidence band (see appendix C); to get a standard uniform distribution, the occurrence times must be scaled to a (0, 1) interval.

b. Marked process

The model requires the iid of the random vectors defining the marks and the mutual independence of the occurrence and the marked processes. These properties are checked using the tools described in sections Aa(2) and Aa(3) (except the trend tests specific for PP). The presence of seasonal behavior is also controlled by drawing a bubble plot for each severity variable; in this plot, droughts are displayed versus their month of occurrence (coded from 1 to 12), using bubbles of size proportional to their value for the severity variable. The Kruskal–Wallis test for comparing the median values in different groups is also applied, although the required hypothesis of equal dispersion in the groups is difficult to assess due to the small sample size available.

APPENDIX B

Estimation of the Model

a. Occurrence process

The intensity parameter of a PClP, λ, is equal to the parameter of the exponential distribution of the corresponding recurrence times; consequently, it can be easily estimated from the sample of drought recurrence times. Applying the invariance property, we can obtain the MLE of the expected number of droughts to occur in a given period of time T, λT, and the expected recurrence time, $1/\lambda$.

We use the $100(1 - \alpha)\%$ confidence intervals for λ, suggested by Johnson et al. (1994),

$$\left[\frac{\hat{\lambda}}{2n} x_{2n}^{\alpha/2}, \frac{\hat{\lambda}}{2n} x_{2n}^{1-\alpha/2}\right],$$

where x_{2n}^{α} is the α percentile of a χ_{2n}^2 distribution. Confidence intervals for λT and $1/\lambda$ can be obtained by applying analogous transformations to the interval for λ.

b. Marked process

To find the best probability distributions to model the severity variables, we perform an empirical selection from a wide range of positive distributions (positiveness is the only restriction for the severity variables). Exponential, Weibull, gamma, lognormal, and GP are considered for the deficit and the maximum intensity. For modeling duration, the distributions are shifted to take values greater than or equal to 1, and negative binomial and geometric distributions are also considered.

The selection is based on graphical tools such as qq plots, the comparison of parametric estimations with their nonparametric counterparts, and plots based on the survival function. More objective procedures such as the likelihood ratio test are applied to compare nested distributions (exponential-Weibull, exponential-gamma, exponential-GP, and geometric-negative binomial). For a goodness of fit analysis, KS tests are available for many distributions. Moreover, we suggest another control that can be applied to any distribution: if the distribution is adequate, the transformation of the sample by the estimated distribution function, $\hat{F}(x_i)$, must show a uniform behavior. Again, uniformity can be checked by the uniform KS test and a uniform qqplot with beta confidence bands (appendix C). See Johnson et al. (1994) for specific properties of the distributions and Coles (2001) for general properties of the MLE.

c. Calculation of confidence intervals

Intervals for the exponential parameter λ and functions of it can be obtained as explained in the occurrence process. For the other distributions, we make use of the properties of the MLE and confidence intervals, for their parameters are obtained using the asymptotic normality and estimating the variances from the observed information matrix. Bootstrap methods (Davison and Hinkley 1997) are used to provide confidence intervals for the mean, percentiles, and other statistics; more precisely, we calculate the basic bootstrap percentile intervals based on 999 generated samples. Another possibility was to calculate asymptotic intervals approximating the variance by the delta method, but in this case bootstrap intervals have been preferred since they are more precise.

APPENDIX C

Qq Plot and Confidence Band for Uniform Samples

Given n iid uniform random variables $\{U_i, i = 1, \ldots, n\}$, the kth ordered statistic $U_{[k]}$ follows a Beta$(k, n - k + 1)$ distribution with mean,

$$E[U_{(k)}] = \frac{k}{n+1}; \qquad \text{(C1)}$$

the quantiles of this distribution can be obtained from a software like S-Plus, for example. The uniform qq plot, where the points $[k/(n + 1), u_{[k]}]$ are represented, can be complemented by plotting an envelope with lower and upper points of coordinates $[k/(n+1), q_k^n(\alpha/2)]$ and $[k/(n+1), q_k^n(1-\alpha/2)]$, respectively, where $q_k^n(\alpha/2)$ and $q_k^n(1-\alpha/2)$ denote the $\alpha/2$ and $(1-\alpha/2)$ quantiles of a Beta$(k, n-k+1)$ distribution. Under the null hypothesis, the n points should approximately describe a line and about $(1-\alpha) \times 100\%$ of them should be contained by the envelope.

REFERENCES

Abaurrea, J., and A. C. Cebrián, 2001: Trend and variability analysis of rainfall series and their extreme events. *Detecting and Modelling Regional Climate Change,* M. Brunet India and D. López Bonillo, Eds., Springer-Verlag, 191–201.

——, and ——, 2002: Drought analysis based on a cluster Poisson model: Distribution of the most severe drought. *Climate Res.,* **22,** 227–235.

Adamson, P. T., A. V. Metcalfe, and B. Parmentier, 1999: Bivariate extreme value distributions: An application of the Gibbs sampler to the analysis of floods. *Water Resour. Res.,* **35,** 2825–2832.

Castro, J. I., and V. Pérez-Abreu, 1994: Some statistical analysis of the cluster point processes of hurricanes on the coasts of Mexico. *Statistics for the Environment 2: Water-Related Issues,* V. Barnett and K. F. Turkman, Eds., Wiley, 95–108.

Coles, S., 2001: *An Introduction to Statistical Modeling of Extreme Values.* Springer-Verlag, 208 pp.

Cox, D. R., and V. Isham, 1980: *Point Processes.* Chapman and Hall, 188 pp.

Davison, A. C., and R. L. Smith, 1990: Models for exceedances over high thresholds. *J. Roy. Stat. Soc.,* **52B,** 393–442.

——, and D. V. Hinkley, 1997: *Bootstrap Methods and Their Application.* Cambridge University Press, 592 pp.

Embrechts, P., C. Kluppelberg, and T. Mikosch, 1997: *Modelling Extremal Events for Insurance and Finance.* Springer-Verlag, 645 pp.

Ferro, C. A. T., and J. Segers, 2003: Inference for clusters of extreme values. *J. Roy. Stat. Soc.,* **65B,** 545–556.

Foley, J. C., 1957: Drought in Australia: Review of records from earliest years of settlement to 1955. Australian Bureau of Meteorology Bulletin 43, 281 pp.

Gertensgarbe, F. W., and P. C. Werner, 1989: A method for the statistical definition of extreme-value regions and their application to meteorological time series. *Z. Meteor.,* **39,** 224–226.

Gibbons, J. D., and S. Chakraborti, 1992: *Nonparametric Statistical Inference.* Dekker, 544 pp.

Gibbs, W. J., and J. V. Maher, 1967: Rainfall deciles as drought indicators. Australian Bureau of Meteorology Bulletin 48, 33 pp.

González-Rouco, J. F., J. L. Jiménez, V. Quesada, and F. Valero, 2001: Quality control and homogeneity of precipitation data in the southwest of Europe. *J. Climate,* **14,** 964–978.

Hisdal, H., and L. M. Tallaksen, 2003: Estimation or regional meteorological and hydrological drought characteristics: A case study for Denmark. *J. Hydrol.,* **281,** 230–247.

Johnson, N. L., S. Kotz, and N. Balakrishnan, 1994: *Continuous Univariate Distributions.* Vols. 1 and 2, 2d ed. Wiley-Interscience, 1475 pp.

Katz, R. W., M. B. Parlange, and P. Naveau, 2002: Statistics of extremes in hydrology. *Adv. Water Resour.,* **25,** 1287–1304.

Kendall, D. R., and J. A. Dracup, 1992: On the generation of drought events using an alternating renewal-reward model. *Stochastic Hydrol. Hydraul.,* **6,** 55–68.

Leadbetter, M. R., G. Lindgren, and H. Rootzén, 1983: *Extremes and Related Properties of Random Sequences and Processes.* Springer-Verlag, 336 pp.

Madsen, H., and D. Rosbjerg, 1998: A regional Bayesian method for estimation of extreme streamflow droughts. *Statistical and Bayesian Methods in Hydrological Sciences,* E. Parent et al., Eds., IHP-V Technical Document in Hydrology, Vol. 20, UNESCO, 327–339.

Marachi, G., 2000: Agricultural drought—A practical approach to definition, assesment and mitigation strategies. *Drought and Drought Mitigation in Europe,* J. V. Vogt and F. Somma, Eds., Kluwer Academic, 63–75.

McKee, T. B., N. J. Doesken, and J. Kleist, 1993: The relationship of drought frequency and duration to time scales. Preprints, *Eighth Conf. on Applied Climatology,* Anaheim, CA, Amer. Meteor. Soc., 179–184.

Meeker, W. Q., and L. A. Escobar, 1998: *Statistical Methods for Reliability Data.* Wiley, 680 pp.

Moyé, L. A., A. S. Kapadia, and I. M. Cech, 1988: The theory of runs with application to drought prediction. *J. Hydrol.,* **103,** 127–137.

Rasmussen, P. F., F. Ashkar, D. Rosbjerg, and B. Bobée, 1994: The POT method for flood estimation: A review. *Extreme Values: Floods and Droughts,* K. W. Hipel, Ed., Vol. 1, Kluwer, 15–26.

Sen, Z., 1976: Wet and dry periods of annual flow series. *J. Hydrol.,* **35,** 311–324.

Tallaksen, L. M., H. Madsen, and B. Clausen, 1997: On the definition and modelling of streamflow drought duration and deficit volume. *Hydrol. Sci.,* **42,** 15–33.

Zelenhasic, E., and A. Salvai, 1987: A method of streamflow analysis. *Water Resour. Res.,* **23,** 156–168.